

C. U. SHAH UNIVERSITY

Summer Examination-2020

Subject Name: Mathematics-I

Subject Code: 4SC01MAT1

Branch: B.Sc. (All)

Semester : 1

Date : 28/02/2020

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

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- Q-1 Attempt the following questions: (14)**
- a) State Rolle's theorem. 02
- b) Find Maclaurin's series of $f(x) = e^x$. 02
- c) Define: Scalar matrix. 01
- d) Write down necessary and sufficient condition for the differential equation to be exact. 01
- e) $(x^2 + y^2)dx - 2xydy = 0$ is differential equation of type _____ 01
(a) Homogeneous (b) Bernoulli's (c) Exact (d) Linear
- f) Write form of Clairaut's equation. 01
- g) $\lim_{x \rightarrow 0} \frac{\tan x}{x}$ is of the form 01
(a) $\frac{\infty}{\infty}$ (b) $\infty - \infty$ (c) $\frac{0}{0}$ (d) 0^0
- h) Find the order of differential equation 01
 $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$.
- i) State Caley-Hamilton theorem 01
- j) The integrating factor of the differential equation
 $ydx - xdy = 0$ is 01
(a) $\frac{1}{x}$ (b) $\frac{1}{y^2}$ (c) $\frac{1}{y}$ (d) None of these
- k) Polar form of equation $x^2 + y^2 = 6xy$ is 01
(a) $1 = 3 \sin 2\theta$ (b) $1 = 2 \sin 3\theta$ (c) $1 = 3 \cos 2\theta$ (d) None
- l) Write down Maclaurin's series of $\cos x$. 01



Attempt any four questions from Q-2 to Q-8

- Q-2 Attempt all questions (14)**
- a) State and Leibnitz's theorem. 07
- b) Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x}$. 05
- c) State Lagrange's mean value theorem. 02
- Q-3 Attempt all questions (14)**
- a) Verify Lagrange's mean value theorem for the function, 06
 $f(x) = (x - 1)(x - 2)(x - 3), \forall x \in [1,4]$.
- b) Solve: $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$ 06
- c) State Taylor's series 02
- Q-4 Attempt all questions (14)**
- a) If $y = (\sin^{-1} x)^2$, show that 06
 $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n$, hence find $(y_n)_0$.
- b) For $x > 0$, show that $\frac{x}{1+x^2} < \tan^{-1} x < x$. 05
- c) Evaluate $\lim_{x \rightarrow 0} \frac{x \cos x - \log(1+x)}{x^2}$. 03
- Q-5 Attempt all questions (14)**
- a) If $(2, \frac{\pi}{4}, \frac{\pi}{6})$ are spherical co-ordinates for a point then find cartesian 06
 and cylindrical co-ordinates.
- b) Verify Cauchy's mean value theorem for the function 05
 $f(x) = \sin x, g(x) = \cos x, \forall x \in [0, \frac{\pi}{2}]$.
- c) Convert the equation $x^3 = y^2(2a - x)$ to the polar equation. 03
- Q-6 Attempt all questions (14)**
- a) Reduce the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ into its normal form 06
 and hence find its rank.
- b) Find the A^{-1} of $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ by Gauss-Jordan method 06
- c) Evaluate: $\begin{vmatrix} 2 & -1 & 1 \\ 3 & -2 & 1 \\ 9 & -5 & 4 \end{vmatrix}$ 02
- Q-7 Attempt all questions (14)**
- a) Verify Caley-Hamilton theorem for the matrix 07



$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

- b) Solve given system of equation by Gauss elimination method 05

$$x + y + 2z = 9$$

$$2x + 4y - 3z = 1$$

$$3x + 6y - 5z = 0$$

- c) Find the equation of sphere for which $A(2, -3, 4)$ and $B(-2, 3, -4)$ are the extremities of a diameter. 02

Q-8 Attempt all questions (14)

- a) Find eigenvalues and eigenvectors of the matrix 05

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

- b) Solve: $\frac{dy}{dx} = x^3 - 2xy$. 05

- c) Find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0$, at the point $(1, 1, -1)$ and passes through the origin. 04

